CERTAIN PROPERTIES OF DIFFUSION FLOWS NEAR THE SURFACE OF AN EVAPORATING DROP

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This article gives asymptotic expressions for the specific diffusion flows at the surface of an evaporating drop and studies some of their properties.

Vaporization and condensational growth of drops are known to be substantially non-steady-state processes [1, 2]. It is therefore of definite interest to investigate those features of solution of the drop-evaporation problem that are associated with consideration of the non-steady-state character of the process and of the mobility of the interface between the liquid and gaseous phases. Certain properties of solutions to non-steady-state problems were considered previously [3, 4].

The solutions of the equations describing quasi-steady-state and non-steady-state evaporation of a water drop, with and without the influence of reduction of its size taken into account through a function characterizing the process rate, can be represented in the form of the following asymptotic series for powers of the small dimensionless parameter Ko:

$$R = R_0 + \mathrm{Ko}\,\frac{D(s-1)}{R_0}\,t - \mathrm{Ko}^2\,\frac{D^2(s-1)^2}{2R_0^3}\,t^2 + \mathrm{Ko}^3\,\frac{D^3(s-1)^3}{2R_0^5}\,t^3 - \ldots, \tag{1}$$

$$R^* = R_0 + \mathrm{Ko}\left[\frac{D(s-1)}{R_0}\,t + \frac{2\sqrt{D}(s-1)}{\sqrt{\pi}}\sqrt{t}\right] - \mathrm{Ko}^2\left[\frac{D^2(s-1)^2}{2R_0^3}\,t^2 + \frac{4D\sqrt{D}(s-1)^2}{3R_0^2\sqrt{\pi}}\,t\sqrt{t}\right] + \ldots, \tag{2}$$

$$R^{**} = R_0 + \mathrm{Ko}\left[\frac{D(s-1)}{R_0}\,t + \frac{2\sqrt{D}(s-1)}{\sqrt{\pi}}\sqrt{t}\right] - \mathrm{Ko}^2\left[\frac{D^2(s-1)^2}{2R_0^3}\,t^2 + \frac{2D\sqrt{D}(s-1)^2}{3R_0^2\sqrt{\pi}}\,t\sqrt{t} - \frac{3\pi-4}{2\pi}\,\frac{D(s-1)^2}{R_0}\,t - \frac{4\sqrt{D}(s-1)^2}{\pi\sqrt{\pi}}\sqrt{t}\right] + \ldots \tag{3}$$

The present article, which is a continuation of earlier studies [3, 4], is devoted to certain properties of the specific diffusion flows near the surface of a drop under quasi-steady-state and non-steady-state evaporation regimes.

Using the equation

$$\frac{dR}{dt} = \frac{D}{\gamma}\,\frac{\partial\rho}{\partial r}\bigg|_{r=R},$$

which defines the rate of change of the drop radius with time, the expressions for the flows are written in the form:

$$j = -\gamma\frac{dR}{dt}, \tag{4}$$

$$j^* = -\gamma\frac{dR^*}{dt}, \tag{5}$$

$$j^{**} = -\gamma\frac{dR^{**}}{dt}. \tag{6}$$

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Differentiating the expansions in Eqs. (1), (2), and (3) for the variable t and substituting the values of the derivatives dR/dt, dR*/dt, and dR**/dt into Eqs. (4), (5), and (6) respectively, we obtain the following relationships for the specific diffusion flows near the drop surface:

$$j = \gamma \mathrm{Ko} \frac{D(1-s)}{R_0} \left[1 + \mathrm{Ko} \frac{D(1-s)}{R_0^2} t + \mathrm{Ko}^2 \frac{3D^2(1-s)^2}{2R_0^4} t^2 + \ldots \right], \tag{7}$$

$$j^* = \gamma \mathrm{Ko} \frac{D(1-s)}{R_0} \left\{ 1 + \frac{R_0}{\sqrt{\pi D t}} + \mathrm{Ko} \left[\frac{D(1-s)}{R_0^2} t + \frac{2\sqrt{D}(1-s)}{R_0 \sqrt{\pi}} \sqrt{t} \right] + \ldots \right\}, \tag{8}$$

$$j^{**} = \gamma \mathrm{Ko} \frac{D(1-s)}{R_0} \left\{ 1 + \frac{R_0}{\sqrt{\pi D t}} + \mathrm{Ko} \left[\frac{D(1-s)}{R_0^2} t \right.\right.$$

$$\left.\left. + \frac{\sqrt{D}(1-s)}{R_0 \sqrt{\pi}} \sqrt{t} - \frac{(3\pi - 4)(1-s)}{2\pi} - \frac{2(1-s)R_0}{\pi \sqrt{\pi D t}} \right] + \ldots \right\}. \tag{9}$$

It is readily seen from Eqs. (7), (8), and (9) that the non-steady-state diffusion flows j* and j** can be represented in the form of the sum of the corresponding quasi-steady-state flow j and certain auxiliary terms that take into account the non-steady-state character of the process, as well as the influence of reduction of drop size (in the form of a function characterizing the evaporation rate).

Using the expansions in Eqs. (7), (8), and (9), we readily obtain the following asymptotic formulas for the functions $j = j(t)$, $j^* = j^*(t)$, and $j^{**} = j^{**}(t)$ when $t \to 0$:

$$j \approx \gamma \mathrm{Ko} \frac{D(1-s)}{R_0},$$

$$j^* \approx \gamma \mathrm{Ko} \frac{D(1-s)}{R_0} + \gamma \mathrm{Ko} \frac{\sqrt{D}(1-s)}{\sqrt{\pi t}},$$

$$j^{**} = \gamma \mathrm{Ko} \frac{D(1-s)}{R_0} + \gamma \mathrm{Ko} \frac{\sqrt{D}(1-s)}{\sqrt{\pi t}} \left(1 - 2\mathrm{Ko} \frac{1-s}{\pi} \right),$$

comparison of which yields

$$j^* - j \approx \gamma \mathrm{Ko} \frac{\sqrt{D}(1-s)}{\sqrt{\pi t}}, \tag{10}$$

$$j^{**} - j \approx \gamma \mathrm{Ko} \frac{\sqrt{D}(1-s)}{\sqrt{\pi t}} \left(1 - 2\mathrm{Ko} \frac{1-s}{\pi} \right), \tag{11}$$

$$j^* - j^{**} \approx \gamma \mathrm{Ko}^2 \frac{2\sqrt{D}(1-s)^2}{\pi \sqrt{\pi t}}. \tag{12}$$

Analysis of series (7), (8), and (9) and asymptotic relationships (10), (11), and (12) leads to the conclusion that the expressions for the non-steady-state diffusion flows differ materially from the corresponding expression for the quasi-steady-state flow near the surface of an evaporating drop during the earliest phases of the process. Moreover, it follows from Eqs. (10), (11), and (12) that the inequality

$$j^* > j^{**} > j \tag{13}$$

is satisfied at $s < 1$ (drop evaporation) and $t \to 0$, which indicates that the non-steady-state diffusion flows from the surface of the drop during evaporation are greater than the corresponding quasi-steady-state flow at times close to the start of the process. Direct comparison of Eqs. (7), (8), and (9) shows that Eq. (13) is satisfied for any value of the variable t in the interval $0 \le t \le t_c$, where t_c is the time required for complete evaporation of the drop, determined from the Maxwell – Sreznevskii formula.

Examining the difference between the expressions for the non-steady-state diffusion flows obtained with and without the mobility of the interface between the liquid and gaseous phases taken into account, it is easily seen that this difference becomes quite large during the initial phase of the process. It also follows from Eqs. (10), (11), and (12) that, with any fixed value of the variable t as close to zero as desired, the differences $j^* - j$ and $j^{**} - j$ are proportional to the first power of the parameter Ko (the term $-2\mathrm{Ko} \times (1-s)/\pi$ in Eq. (11) is neglected, since it is much less than one), while the difference $j^* - j^{**}$ is proportional to Ko^2. It is therefore not important to take into account the influence of the decrease in drop size (in the form of a function describing the evaporation rate), which is small in comparison with the influence of the non-quasi-steady-state character of the process during its initial phase.

From the mathematical standpoint, the above differences in the asymptotic properties of the expressions for the flows j* and j and the flows j** and j at times close to the start of evaporation can be attributed to the fact that the derivatives dR^*/dt and dR^{**}/dt tend to $-\infty$ when $t \to 0$, so that $\lim j^* = +\infty$ and $\lim j^{**} = +\infty$, while the derivative dR/dt has a limit as $t \to 0$ that is constant and equals $-\mathrm{Ko}[D(1-s)/R_0]$ and $\lim j = \gamma\mathrm{Ko}[D(1-s)/R_0]$. This asymptotic behavior of the expressions for the specific diffusion flows j* and j** when $t \to 0$ is obviously due to the fact that, during non-steady-state evaporation (both with and without the mobility of the interface between the liquid and gaseous phases taken into account), there is a discontinuity in the density distribution for the vapor in the immediate vicinity of the drop during the initial period of the process; a transition to a continuous distribution occurs later.

If condensation of vapor from the gaseous phase occurs at the drop surface, the expressions for the specific diffusion flows near the drop surface equal the right sides of the asymptotic expansions (7), (8), and (9) with opposite signs. Analysis of these expressions shows that the inequality

$$j^{**} > j^* > j$$

holds for condensational drop growth, being satisfied at any t in the interval $0 \le t \le t_c$.

Consideration of the specific material flows near the drop surface during condensation on it of vapor from the surrounding space under a quasi-steady-state or nonsteady-state regime (with or without taking into account the influence of the change in drop size through a function characterizing the rate of condensational growth) leads to conclusions analogous to those drawn in investigating drop evaporation (in the sense of the asymptotic behavior of the expressions for the specific diffusion flows near the drop surface at the start of the process).

The results obtained above on the asymptotic behavior of the expressions for the specific diffusion flows near the surface of an evaporating drop (or a drop on whose surface vapor is condensing from the surrounding space) very early in the process are in agreement with the conclusions of Todes [5].

It must be noted that solution of the quasi-steady-state and non-steady-state drop-evaporation problems, represented in the form of series for powers of a small dimensionless parameter (equal to the ratio of the density of the saturated vapor at the drop temperature to the density of the drop material), is very convenient for studying the basic characteristics of evaporation, which involves taking into account the influence of the decrease in drop size through a function describing the process rate, and for examining the possibility of using the quasi-steady-state approximation in the theory of drop evaporation and growth. A separate report will be devoted to discussion of this problem.

NOTATION

R, R*, R**	are the respective drop radii during quasi-steady-state evaporation, non-steady-state evaporation with mobility of liquid–gas interface neglected, and non-steady-state evaporation with interface mobility taken into account;
t	is the time;
Ko	is the dimensionless parameter equal to ratio of density of saturated vapor at drop temperature to density of drop material;
D	is the diffusion constant of vapor in air;
s	is the supersaturation;
R_0	is the initial drop radius;
γ	is the density of drop material;
ρ	is the density of vapor in space surrounding drop;
r	is the spatial coordinate;
j, j*, j**	are the specific diffusion flows near surface of drops with radii R, R*, and R** respectively.

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